HEAT TRANSFER WITH TWO-DIMENSIONAL LIQUID FILTRATION INTO SHATTERED ROCKS

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An analytic solution is obtained for problems concerning heat transfer by the filtration of a heat-transfer agent into shattered rocks, and a comparison is carried out between the calculation results for different models of the heat-transfer process.

One of the principal problems when investigating thermal processes in underground thermal boilers (UTB) is the establishment of the mechanisms of the interphase heat transfer during the liquid filtration into the shattered rocks.

An estimate of the error in calculating the coolant temperature, due to nonuniformity of the filtration flow and by the assumption that the interphase heat-transfer coefficient is equal to its quasi-steady-state value, can be determined by comparing the solutions of problems in which the non-steady-state heat transfer in an underground thermal boiler is described by taking account of the temperature gradient in blocks of rock and with the assumption of linearity of the interphase heat-transfer [1].

1. We shall represent the structural model of the underground thermal boiler in the form of a purely fractured medium, consisting of impermeable blocks of rock with a regular geometric shape and identical dimensions with a regular packing. In the space between the rocks, determined by the magnitude of the fracture porosity, a liquid is moving in the directions, and with velocities, governed by the distribution of the pressure gradient and of the fracture permeability of the medium.

We shall assume that one-dimensional conductive heat transfer takes place in the blocks, in a direction normal to their surfaces, and that the thermal resistance at the rock and liquid interface is small by comparison with the thermal resistance of the blocks of rock.

If we take into account that for the conditions of the underground thermal boiler, the effect of nonisothermal filtration of the coolant on its mass velocity can be neglected, the mathematical formulation of the problem will have the form

$$\lambda_\Gamma \frac{1}{z^p}\cdot\frac{\partial}{\partial z}\left(z^p\frac{\partial T}{\partial z}\right) = c_\Gamma\rho_\Gamma\frac{\partial T}{\partial \tau},\quad 0<z<\frac{a}{2}; \tag{1}$$

$$-c_l\,\mathbf{W}\,\mathrm{grad}\,T - \lambda_\Gamma S_v\frac{\partial T}{\partial z} = c_l\rho_l m\frac{\partial T}{\partial \tau},\quad z=\frac{a}{2}; \tag{2}$$

$$\mathrm{div}\,\mathbf{W} = 0; \tag{3}$$

$$T|_{\Gamma_i,\,z=a/2} = T_b; \tag{4}$$

$$T|_{\tau=0} = T_0; \tag{5}$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=0} = 0; \tag{6}$$

$$G_k = H\int_{\Gamma_k}(\mathbf{W}\cdot\mathbf{l})\,d\Gamma_k^*, \tag{7}$$

where

$$\mathbf{W} = W_x\mathbf{i} + W_y\mathbf{j};\quad \mathrm{grad}\,T = \frac{\partial T}{\partial x}\mathbf{i} + \frac{\partial T}{\partial y}\mathbf{j};$$

$$T_l = T_\Gamma|_{z=a/2};\quad k = 1, 2, \ldots, n.$$

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Suppose that W_x and W_y are the solution of Eq. (3) with the condition (7), and G_k is independent of τ when $1 \leq k \leq n$.

We shall assume, that the blocks have the shape of plane-parallel unrestricted plates. Then, by introducing the variables $\theta = (T_0 - T)/(T_0 - T_b)$, $\nu = 2z/a$ and $Fo = 4\lambda_r \tau / \rho_r c_r a^2$, and taking account of the smallness of the right-hand side of Eq. (2), we shall have

$$\frac{\partial^2 \theta}{\partial \nu^2} = \frac{\partial \theta}{\partial Fo}, \quad 0 < \nu < 1, \tag{8}$$

$$-\mathbf{W} \operatorname{grad} \theta - B \frac{\partial \theta}{\partial \nu} = kB \frac{\partial \theta}{\partial Fo}, \quad \nu = 1; \tag{9}$$

$$\theta|_{\Gamma_i, \nu=1} = 1; \tag{10}$$

$$\theta|_{Fo=0} = 0; \tag{11}$$

$$\left. \frac{\partial \theta}{\partial \nu} \right|_{\nu=0} = 0, \tag{12}$$

where

$$B = 4\lambda_r / a(a+b) c_l; \quad k = c_l \rho_l^0 b / \rho_r c_r a;$$

$$\rho_l^0 = \rho_l \quad \text{for} \quad T_l = \text{const.}$$

We apply to the problem (8)-(12), the Laplace integral transform:

$$\frac{\partial^2 \bar{\theta}}{\partial \nu^2} = s\bar{\theta}, \quad 0 < \nu < 1; \tag{13}$$

$$-\mathbf{W} \operatorname{grad} \bar{\theta} - B \frac{\partial \bar{\theta}}{\partial \nu} = kBs\bar{\theta}, \quad \nu = 1; \tag{14}$$

$$\bar{\theta}|_{\Gamma_i, \nu=1} = \frac{1}{s}; \tag{15}$$

$$\left. \frac{\partial \bar{\theta}}{\partial \nu} \right|_{\nu=0} = 0. \tag{16}$$

Solving Eq. (13), taking account of condition (16), we obtain

$$\bar{\theta} = A(x, y) \operatorname{ch} \nu \sqrt{s}; \tag{17}$$

with $\nu = 1$

$$\mathbf{W} \operatorname{grad} A = -LA; \tag{18}$$

$$A|_{\Gamma_i} = \frac{1}{s \operatorname{ch} \sqrt{s}}, \tag{19}$$

where $L = B(\sqrt{s} \tanh \sqrt{s} + ks)$.

Converting to ordinary differential equation, we obtain

$$\frac{dx}{W_x} = \frac{dy}{W_y} = -\frac{1}{L} \cdot \frac{dA}{A}, \tag{20}$$

and the first integral of system (20) represents the family of flow lines

$$C = F(x, \gamma x), \tag{21}$$

where $\gamma = y/x$.

Taking into account Eq. (20) and (21), we obtain

$$\frac{d\gamma}{dA} = -\frac{f(C, \gamma)}{AL}, \tag{22}$$

where

$$f(C, \gamma) = \frac{1}{F^{-1}(C, \gamma)} (W_y - \gamma W_x). \tag{23}$$

The solution of Eq. (22), taking account of Eq. (19), has the form

$$A=\frac{1}{s\,\mathrm{ch}\sqrt{s}}\exp\left[-L\int\limits_{\gamma_{r_i}}^{\gamma}\frac{d\gamma^*}{f(C,\gamma^*)}\right]. \tag{24}$$

Then

$$\tilde{\theta}=\frac{\mathrm{ch}\,\nu\sqrt{s}}{s\,\mathrm{ch}\sqrt{s}}\exp\left[-\rho_r(\sqrt{s}\,\mathrm{th}\sqrt{s}+ks)\right], \tag{25}$$

where

$$\rho_r=B\int\limits_{\gamma_{r_i}}^{\gamma}\frac{d\gamma^*}{f(C,\gamma^*)}. \tag{26}$$

Carrying out the inverse transformation [2], with $\nu=1$, we shall have

$$\theta_l=\left\{\frac{1}{2}+\frac{2}{\pi}\sum_{j=1}^{2}\int\limits_0^1\exp\left(-\frac{\rho_r}{2}\psi_j\frac{\mathrm{sh}\,\psi_j-\sin\psi_j}{\mathrm{ch}\,\psi_j+\cos\psi_j}\right)\times\right.$$
$$\left.\times\sin\left[\frac{\psi_j}{2}\left(\eta_r\psi_j-\rho_r\frac{\mathrm{sh}\,\psi_j+\sin\psi_j}{\mathrm{ch}\,\psi_j+\cos\psi_j}\right)\right]\frac{d\psi}{\psi}\right\}U(\eta_r), \tag{27}$$

where

$$\psi_1=\eta_r\psi;\quad \psi_2=\frac{\eta_r}{\psi};\quad U(\eta_r)=\begin{cases}0, & k\rho_r\geqslant\mathrm{Fo}\\ 1, & k\rho_r<\mathrm{Fo}\end{cases};$$
$$\eta_r=\mathrm{Fo}-k\rho_r.$$

The temperature of the coolant at the end face of the production well (at the outlet from the underground thermal boiler) is

$$\theta_c=\frac{H}{G_p}\int\limits_{\Gamma_p}\theta_l(\mathbf{W}\cdot\mathbf{l})\,d\Gamma_p^* \tag{28}$$

2. In the case of linearity of the interphase heat transfer between the rock and the liquid, the mathematical formulation of the problem will have the form

$$\alpha_v(T_l-T_r)=c_r\rho_r(1-m)\frac{\partial T_r}{\partial\tau}; \tag{29}$$

$$-\mathbf{W}c_l\,\mathrm{grad}\,T_l+\alpha_v(T_r-T_l)=c_l\rho_l^0 m\frac{\partial T_l}{\partial\tau}; \tag{30}$$

$$\mathrm{div}\,\mathbf{W}=0; \tag{31}$$

$$T_l|_{r_i}=T_b; \tag{32}$$

$$T_l|_{\tau=0}=T_r|_{\tau=0}=T_0; \tag{33}$$

$$G_k=H\int\limits_{\Gamma_k}(\mathbf{W}\cdot\mathbf{l})\,d\Gamma_k^*. \tag{34}$$

We introduce the variables θ_r, θ_l and $\xi=\alpha_v\tau/c_r\rho_r(1-m)$ and apply the Laplace integral transform to the problem:

$$(1+s)\tilde{\theta}_r-\tilde{\theta}_l=0; \tag{35}$$

$$-\mathbf{W}\,\mathrm{grad}\,\tilde{\theta}_l+B_1\tilde{\theta}_r-B_1(1+ks)\tilde{\theta}_l=0; \tag{36}$$

$$\tilde{\theta}_l|_{r_i}=\frac{1}{s}, \tag{37}$$

where

$$B_1=\alpha_v/c_l.$$

The solution of Eq. (35)-(37), has the form

$$\tilde{\theta}_l = \frac{1}{s}\exp\left[-L_1\int\limits_{\gamma_{r_i}}^{\gamma}\frac{d\gamma^*}{f(C,\gamma^*)}\right]; \quad (38)$$

$$\tilde{\theta}_r = \frac{1}{s(1+s)}\exp\left[-L_1\int\limits_{\gamma_{r_i}}^{\gamma}\frac{d\gamma^*}{f(C,\gamma^*)}\right], \quad (39)$$

where

$$L_1 = B_1 s(1+k+ks)/(1+s).$$

Carrying out the inverse transformation [1], we obtain

$$\theta_l = \exp(-\rho)\Big[\exp(-\eta)I_0(2\sqrt{\rho\eta}) + \int\limits_0^{\eta}\exp(-\eta^*)I_0(2\sqrt{\rho\eta^*})\,d\eta^*\Big]U(\eta); \quad (40)$$

$$\theta_r = \exp(-\rho)\int\limits_0^{\eta}\exp(-\eta^*)I_0(2\sqrt{\rho\eta^*})\,d\eta^*\,U(\eta), \quad (41)$$

where

$$\eta = \xi - k\rho; \quad U(\eta) = \begin{cases}0, & k\rho \geqslant \xi,\\ 1, & k\rho < \xi,\end{cases}$$

$$\rho = B_1\int\limits_{\gamma_{r_i}}^{\gamma}\frac{d\gamma^*}{f(C,\gamma^*)}. \quad (42)$$

When $\alpha_v \to \infty$ (thermally uniform medium), the position of the convective temperature front is

$$\theta_l = \begin{cases}0, & \tau \leqslant \tau^*,\\ 1, & \tau > \tau^*,\end{cases} \quad (43)$$

where

$$\tau^* = \frac{c_m\rho_m}{c_l}\int\limits_{\gamma_{r_i}}^{\gamma}\frac{d\gamma^*}{f(C,\gamma^*)}, \quad (44)$$

$$c_m\rho_m = c_r\rho_r(1-m) + c_l\rho_l^0 m.$$

We note that $\theta_c = 0$ when $\tau < \tau_0$, and the time of convergence of the convective temperature front at the contour Γ_p is

$$\tau_0 = \frac{\pi R^2 H c_m \rho_m k_m}{G_p c_l},$$

and k_M is determined by the results of [3].

Taking into account the assumptions made, the relations obtained permit the temperature of the coolant to be determined at any point of the region and at the outlet from the underground thermal boiler for different models of the heat-transfer process, if function (23) is known, depending on the filtration-rate distribution of the coolant.

3. We shall consider a few special cases:

a) With rectilinear filtration of the liquid (W_x = const), the flow line equation is $y = C$.

Then

$$f(C,\gamma) = -\frac{\gamma^2}{C}W_x; \quad \rho = \frac{\alpha_v x}{c_l W_x};$$

$$\int\limits_{\gamma_{r_i}}^{\gamma}\frac{d\gamma^*}{f(C,\gamma^*)} = \frac{x}{W_x}; \quad \rho_r = \frac{4\lambda_r x}{a(a+b)c_l W_x},$$

which coincides with [2].

b) With plane-radial filtration of the liquid ($W_{pr} = G/2\pi r$; $r^2 = x^2 + y^2$)

$$\rho = \frac{\pi H \alpha_v (r^2 - r_i^2)}{c_l G}\ ; \quad \rho r = \frac{4\pi H \lambda_r (r^2 - r_i^2)}{a(a+b) c_l G}\ .$$

c) In the case of filtration of the coolant from two injection boreholes, for which the coordinates of the axes are $x = \pm R$; $y = 0$, to the production well ($x = y = 0$), the boundary conditions have the form

$$W_{pr}|_{r=r_p} = \frac{G}{2\pi H r_c}\ ; \quad W_{pr}|_{r=r_i} = \frac{G}{4\pi H r_c}\ , \tag{45}$$

and $r_p = r_i = r_c$.

If the mass filtration-rate potential satisfies the Laplace equation, the solution of Eq. (3) with conditions (45) can be represented in the form

$$W_x = \frac{G}{2\pi H}\left[\frac{x(x^2 + y^2 - R^2)}{(x^2 + y^2 + R^2)^2 - (2Rx)^2} - \frac{x}{x^2 + y^2}\right];$$

$$W_y = \frac{G}{2\pi H}\left[\frac{y(x^2 + y^2 + R^2)}{(x^2 + y^2 + R^2)^2 - (2Rx)^2} - \frac{y}{x^2 + y^2}\right].$$

As

$$C = \frac{xy}{(x^2 + y^2)^2 + R^2(y^2 - x^2)}\ ,$$

then

$$f(C,\ \gamma) = \frac{GC^2R^2(1+\gamma^2)^2}{\pi H \gamma (1 + 4R^4C^2)}\ .$$

Using Eq. (26), (42) and (44), we obtain

$$\rho = \frac{\pi H \alpha_v \beta}{G c_l}\ ; \tag{46}$$

$$\rho r = \frac{4\pi H \lambda_r \beta}{G c_l a(a+b)}\ ; \tag{47}$$

$$\tau^* = \frac{\pi H c_m \rho_m \beta}{G c_l}, \tag{48}$$

where

$$\beta = \frac{\gamma^2(1 + 4R^4C^2)}{2C^2R^2(1+\gamma^2)}\ . \tag{49}$$

As

$$k_m = \lim_{\gamma \to 0} \frac{\beta}{R^2} = \frac{(R^2 - r_c^2)^2}{2R^4}\ ,$$

then for $r_c \ll R$

$$\tau_0 = \frac{\pi R^2 H c_m \rho_m}{2 G c_l}. \tag{50}$$

d) In the case of filtration of the coolant from an injection borehole ($x = R$; $y = 0$) and the production well ($x = y = 0$)

$$W_{pr}|_{r=r_c} = \frac{G}{2\pi H r_c}\ . \tag{51}$$

The solution of Eq. (3) with the condition (51) is

$$W_x = \frac{G}{2\pi H}\left[\frac{x - R}{(R - x)^2 + y^2} - \frac{x}{x^2 + y^2}\right];$$

$$W_y = \frac{G}{2\pi H}\left[\frac{y}{(R - x)^2 + y^2} - \frac{y}{x^2 + y^2}\right].$$

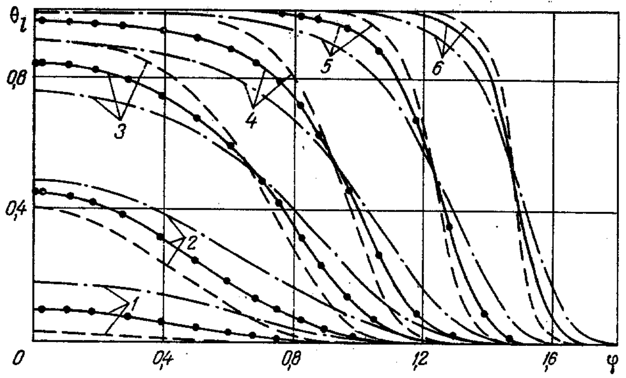

Fig. 1. Change of θ_l versus φ on Γ_{pw}. The dashed lines are ρ_0=94.96 with ξ; 1) 68.0, 2) 91.22, 3) 114.83, 4) 136.64, 5) 182.45, 6) 250.87. Solid lines are ρ_0=46.52 for ξ; 1) 33.52, 2) 44.70, 3) 55.87, 4) 67.05, 5) 89.40, 6) 122.92. Dash–dot lines are ρ_0=20.67 with ξ: 1) 14.90, 2) 19.87, 3) 24.83, 4) 29.80, 5) 39.73, 6) 54.63 [calculation by Eq. (40)]. Points are a calculation by Eq. (27); φ, deg.

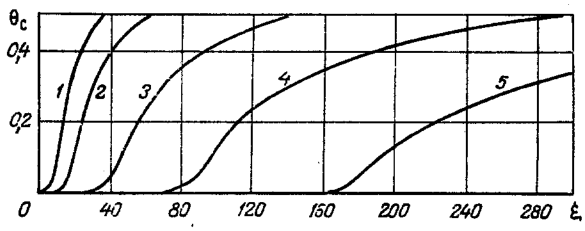

Fig. 2

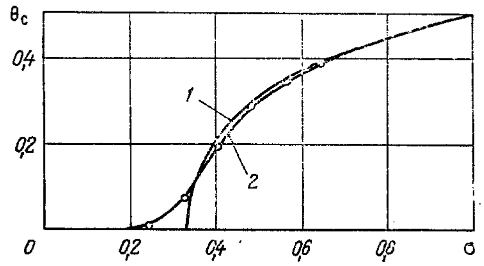

Fig. 3

Fig. 2. Change of θ_c versus ξ at the face of the production well by Eq. (53) and (40), for ρ_0: 1) 11.62, 2) 20.67, 3) 46.52, 4) 94.96, 5) 186.07.

Fig. 3. Change of θ_c at the face of the production well versus $\sigma = Gc_l\tau/\pi R^2 Hc_m\rho_m$: 1) Calculation by Eq. (54) and (52); 2) by Eq. (53) and (40); points are a calculation by Eq. (53) and (27).

As

$$f(c,\ \gamma) = \frac{GC^3R(1+\gamma^2)^2}{2\pi H\gamma(\gamma+CR)(1+C^2R^2)},$$

where

$$C = \frac{y}{y^2 + x(x-R)},$$

then ρ, ρ_Γ, and τ^* are determined by relations (46)-(48), where

$$\beta = \frac{1+C^2R^2}{C^2}\left[1 + \frac{1}{CR}\left(\operatorname{arctg}\gamma - \frac{\gamma+CR}{1+\gamma^2}\right)\right]. \qquad (52)$$

The temperature at the end face of the production well Eq. (28) is

$$\theta_c = \frac{R}{\pi}\int\limits_0^\pi \frac{R - r_c\cos\varphi}{R^2 + r_c^2 - 2Rr_c\cos\varphi}\,\theta_l(r_p,\ \varphi)\,d\varphi, \qquad (53)$$

where φ =arctan γ. With $\alpha_\nu \to \infty$

$$\theta_c = \frac{\varphi}{2\pi} + \frac{1}{\pi}\operatorname{arctg}\left(\frac{R+r_c}{R-r_c}\operatorname{tg}\frac{\varphi}{2}\right), \qquad (54)$$

and φ and τ when $\varphi > 0$ are connected by relations (48) and (52). If $r_c \ll R$, then Eq. (54) assumes the form

$$\theta_c = \frac{\varphi}{\pi}.$$

As

$$k_m = \frac{(R - r_c)^2}{R^2} - \frac{2(R - r_c)^3}{3R^3},$$

then for $r_c \ll R$

$$\tau_0 = \frac{\pi R^2 H c_m \rho_m}{3Gc_l}. \quad (55)$$

It can be seen from Figs. 1-3, that if $\theta_l = \theta_l^* \approx 1$ at Γ_p for $\varphi = 0$, then the values of θ_c, determined with a finite value of α_v and for $\alpha_v \to \infty$, almost coincide. Using the simplified solution [4], we obtain the condition

$$-\sqrt{\rho_0} + \sqrt{\eta_0} \geqslant \delta, \quad (56)$$

where

$$\operatorname{erf}\delta = 2\theta_l^* - 1; \quad \rho_0 = \rho|_{\varphi\to 0} = \frac{\pi R^2 H \alpha_v k_m}{Gc_l}; \quad \eta_0 = \xi - k\rho_0.$$

Therefore, if

$$\xi \geqslant \rho_0(1 + k) + \delta^2 + 2\delta\sqrt{\rho_0}$$

or when $k \ll 1$

$$\tau \geqslant \frac{c_r \rho_r}{\alpha_v}\left(R\sqrt{\frac{\pi H \alpha_v k_m}{Gc_l}} + \delta\right)^2,$$

where $\delta \approx 2.2$ when $\theta_l^* = 0.999$, then the values of θ_c can be determined, by using the formulas for a thermally uniform medium.

Comparison of the results of the calculation of θ_l and θ_c (Figs. 1-3) for the models considered of the heat-transfer process between rock and liquid ($\lambda_r = 2.13$ W/m · deg and $a = 10$ m) permit the conclusion that the thermal cycle in the underground thermal boiler is close to quasi-steady-state, and that a model constructed by taking account of the linearity of the interphase heat transfer can be used for its thermal calculation.

NOTATION

i, j, unit vectors of the rectangular-coordinate system; Γ, well contour; l, external normal to Γ; n, number of wells; W, mass filtration rate of coolant; α_v, interphase heat-transfer coefficient; λ, thermal-conductivity coefficient; c, specific heat; ρ, density; S_v, specific surface area or rock blocks; m, fracture porosity; a, linear size of block; H, height of underground thermal boiler; τ, time; T, temperature; R, distance between injection well and production well; b, crack spacing; k_m, minimum value of reduction factor; r, radius of well; $I_0(x)$, modified Bessel function of first kind, zero order; s, complex argument of Laplace integral transform; p = 0, 1, 2 (respectively, for blocks in the shape of plates, cylinders, and spheres). Indices: r, l, i, and p, refer to rocks, liquid, injection, and production wells, respectively.

LITERATURE CITED

1. A. S. Tsyrul'nikov, I. A. Ryzhenko, É. I. Merzlyakov, and V. T. Beloborod'ko, in: Thermophysics and Thermotechniques [in Russian], Naukova Dumka, Issue 27 (1974).
2. E. S. Romm, in: Problems of the Development of the Mineral Deposits of the North [in Russian], Leningrad Mining Institute, Leningrad (1972).
3. É. I. Merzlyakov, in: Problems of Technical Thermophysics [in Russian], Naukova Dumka, No. 5 (1975).
4. I. A. Charnyi, Neft. Khoz., No. 2, 3 (1953).